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The Drell-Yan process and Deep Inelastic Scattering from the lattice[*]

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We report on measurements of the h_1 structure function, relevant to calculating cross-sections for the Drell-Yan process. This is a quantity which can not be measured in Deep Inelastic Scattering, it gives additional information on the spin carried by the valence quarks, as well as insights on how relativistic the quarks are.

1. INTRODUCTION

Deep Inelastic Scattering (DIS) gave us the first evidence that quarks are true physical objects, not just "book-keeping" devices for a flavour symmetry group. The structure functions measured in DIS give us an important insight into the internal workings of a hadron, allowing us to measure how the energy and spin is shared out between the different constituents.

Perturbative QCD can explain the evolution of these structure functions as we change the scale at which we probe the hadron, but is unable to give us a starting value for this evolution. Lattice QCD has been reasonably successful for masses, the next step is to use it to calculate structure function moments, form factors and matrix elements. At present this is our only hope of finding these numbers from first principles.

2. THE INTERPRETATION OF h_1

Deep Inelastic Scattering (DIS) is not the only useful probe of the hadrons' parton distributions.

In hadron-hadron colliders the Drell-Yan process can be observed, in which a quark in one hadron and an anti-quark in the other annihilate to form an extremely virtual time-like photon, which then decays to a lepton-antilepton pair. Measurements of the lepton pair's total momentum give enough information to extract x and y, the fraction each parton carries of its hadron's momentum.

If we look at the unpolarised Drell-Yan process we find the same structure functions occurring as in DIS. For example the total cross-section is proportional to $\sum_a e_a^2 f_1^a(x) f_1^{\bar{a}}(y)$ where a runs over all flavours. The asymmetry in cross-sections when two longitudinally polarised nucleons collide can again be expressed in terms of a quantity known from polarised DIS, namely the structure function g_1. However if we consider the cross-section when two transversely polarised nucleons with spins S_A and S_B collide, and try to find the asymmetry on flipping one spin,

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\begin{aligned}
A_{TT} &\equiv \frac{\sigma(S_A, S_B) - \sigma(S_A, -S_B)}{\sigma(S_A, S_B) + \sigma(S_A, -S_B)} \\
&\propto \frac{\sum_a e_a^2 h_1^a(x) h_1^{\bar{a}}(y)}{\sum_a e_a^2 f_1^a(x) f_1^{\bar{a}}(y)} ,
\end{aligned} \tag{1}
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we find that it can not be expressed in terms of the familiar structure functions, a new structure function, h_1, is needed [1].

Moments of the structure function h_1 can be related, through the operator product expansion, to the matrix elements of the operators

$$O^{\sigma,\mu_1,\cdots,\mu_n} \equiv \bar{\psi} i \sigma^{\sigma\{\mu_1} \gamma_5 iD^{\mu_2} \cdots iD^{\mu_n\}}\psi - \text{tr}. \quad (2)$$

This operator is very similar in structure to the operators that give the moments of f_1 and g_1. An important difference is in the Dirac structure of the operator. In f_1 and g_1 this is proportional to γ_μ or $\gamma_\mu\gamma_5$ respectively, in both cases these matrices anti-commute with γ_5. However the operator in Eq. (2) is proportional to a σ matrix, which commutes with γ_5, implying that h_1 has the opposite chiral properties to f_1 and g_1. It is this difference that explains why h_1 is observable in the Drell-Yan process, but not in DIS. With massless quarks chirality is conserved along a quark line. In the Feynman diagram for DIS (Fig. 1) there is only a single quark line passing through the hard part of the process, so only the chirally-even structure functions f_1 and g_1 can be observed. On the other hand the Drell-Yan process involves two separate fermion lines, which can have the same or opposite chirality. This means that, even at leading twist, both chirally-even and chirally-odd structure functions are observable.
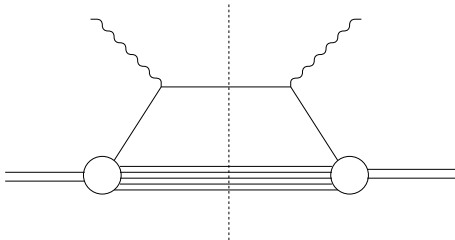

Figure 1. The Feynman diagram for deep inelastic scattering. The hard scattering process conserves chirality.

What does the structure function h_1 tell us about the quarks in the proton? If we consider a stationary proton (momentum in the 0-direction) the operator in Eq. (2) differs from the operator for g_1 by a factor γ_0. In the non-relativistic limit
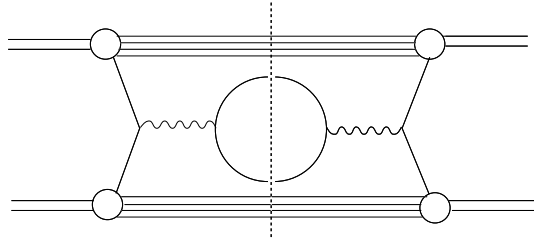

Figure 2. The Feynman diagram for the Drell-Yan process. Because two separate fermion lines are involved, chirality need not be conserved.

fermions are in eigenstates of γ_0 with eigenvalue 1, and so h_1 and g_1 are identical. By comparing h_1 and g_1 for a real proton we can gain an insight into how relativistic the constituents are.

The structure functions h_1 and g_1 have the opposite behaviour under charge conjugation, so it is expected that the contributions of the quarks and anti-quarks in the sea will largely cancel in h_1, making it a quantity given mostly by the valence quarks. This means that we might hope that a quenched calculation of h_1 is likely to give an answer close to the true value.

Beyond the non-relativistic approximation, what does h_1 measure? By considering the effect of operating with the operator from Eq. (2) on a quark state which is an eigenstate of the operator $\slashed{s}_\perp \gamma_5$ [1] it can be seen that what is actually being measured by h_1 is the distribution of this quantity, which is given the name "transversity".

3. RESULTS

The operator in Eq. (2) can be discretised and its expectation value measured on the lattice using the methods used earlier [2] for the more familiar f_1 and g_1 structure functions.

We have undertaken measurements of the quenched QCD structure functions. The measurements I report here were made with the quenched Sheikholeslami-Wohlert action on a $16^3 \times 32$ lattice with $\beta = 6.0$ and $c_{SW} = 1.769$ (the value recommended in [3], chosen to eliminate $O(a)$ discretisation effects). To completely

remove all $O(a)$ effects it is not enough to simply use the optimal fermion action, the lattice operators must also be improved. At present we are using tree-level operator improvement. To produce a final answer we need to know the Z factors of renormalisation theory. We are using the 1-loop perturbative calculation of [4]. Ideally we would like to determine both the operator improvement and the Z factors non-perturbatively, we are currently working on this calculation.
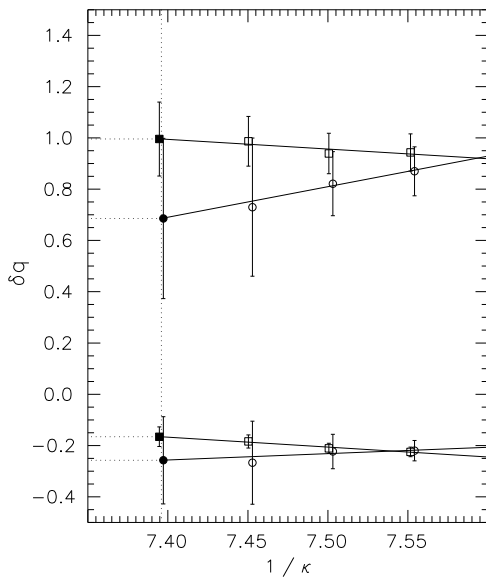

Figure 3. The proton tensor charge $\delta q \equiv \int_0^1 dx\,(h_1(x) - \bar{h}_1(x))$ for the valence u (upper two lines) and d (lower two lines) quarks. The open points are the data, the solid points the chiral extrapolation.

In Fig. 3 we show our results for the proton $\delta q \equiv \int_0^1 dx\,(h_1(x) - \bar{h}_1(x))$ for the u quarks (upper two lines) and d quarks (lower lines). This lowest moment of h_1 is found from the $n = 1$ case of Eq. (2). We have made two determinations of δq, once from the expectation value of the operator $O^{2,4}$ calculated with a stationary nucleon (squares), and once from the operator $O^{1,2}$ measured for a nucleon with one unit of momentum in the 1-direction (circles). (In both cases our nucleon spin is polarised in the 2-direction.) If Lorentz symmetry has been restored on the lattice, both determinations would agree, which they do within the errors, though for $O^{1,2}$ the errors are large. In the heavy quark limit the u and d quark contributions to δq are $+4/3$ and $-1/3$ respectively. We see that the u contribution has dropped below this value for our quark masses.

It is interesting to compare the lattice results with the results of a recent QCD sum rule calculation [5], which finds $\delta u = 1.33 \pm 0.53$ and $\delta d = 0.04 \pm 0.02$ at the scale $1\mathrm{GeV}^2$. The small value of δd differs from the results of lattice gauge theory, (see Fig. 3 or [7]).

As mentioned in the previous section, the comparison between h_1 and g_1 gives us an impression of how relativistic the quarks in a nucleon are. Comparing δq and the valence quark contribution to $\Delta q \equiv \int_0^1 dx g_1(x)$, as reported in [6], we see that they are rather similar, (a conclusion also reached in [7,8]) showing that for the quark masses used, which are approximately in the strange quark range, a non-relativistic description of the spin structure is reasonable.

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